Exercise 5

A cylindrical tank with radius 5 m is being filled with water at a rate of 3 m^3/min. How fast is the height of the water increasing?

Solution

The volume of a cylinder with radius $r = 5$ m and height h is

$$V = \pi r^2 h = \pi(5)^2 h = 25\pi h.$$

Differentiate both sides with respect to t.

$$\frac{d}{dt}(V) = \frac{d}{dt}(25\pi h)$$

$$\frac{dV}{dt} = 25\pi \frac{dh}{dt}$$

Solve for dh/dt, the rate that the height of the water is increasing.

$$\frac{dh}{dt} = \frac{1}{25\pi}\frac{dV}{dt}$$

The tank is being filled at a rate of 3 cubic meters per minute, so $dV/dt = 3$ m^3/min. Therefore, the rate that the height is increasing is

$$\frac{dh}{dt} = \frac{1}{25\pi}(3) = \frac{3}{25\pi}\ \frac{\text{m}}{\text{min}}.$$